PSD Analysis of Fizeau Interferometer Data

Analysis of transmitted wavefront data provided by a commercial supplier

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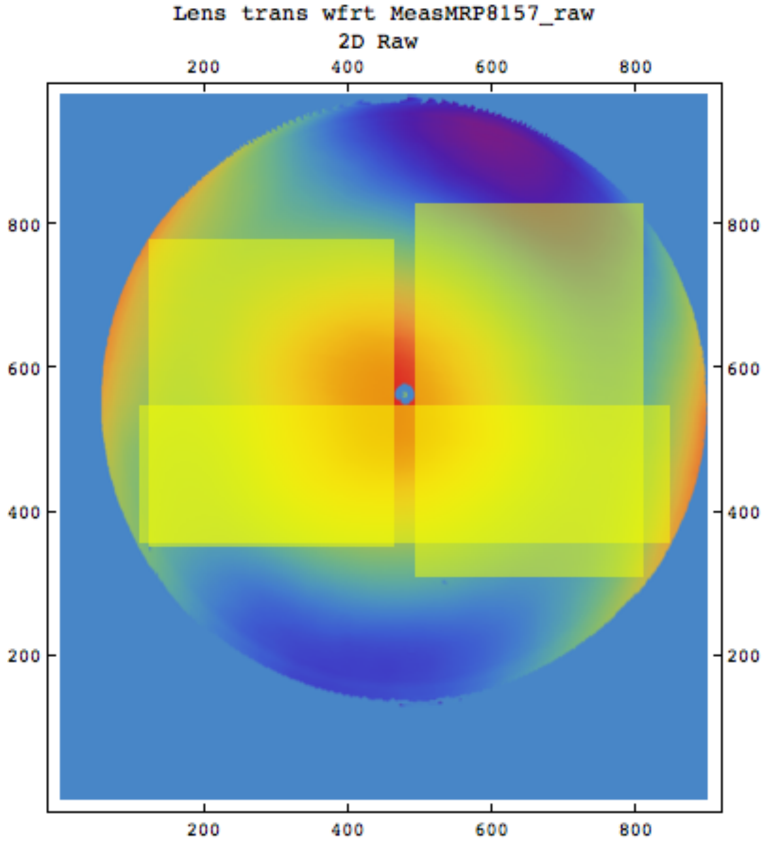
Three ROI regions were extracted and analyzed from the MRP8175_raw.asc data file. The regions were rectangular and excluded the missing points in the center of the aperture. A 2D Cartesian PSD was computed for each region with and without a 2D Blackman window applied. The Cartesian PSD was converted into a radial PSD by azimuthal averaging of points within uniformly spaced radial rings. The 3 resultant rPSDs were then interpolated to a uniform frequency grid and averaged. The problem is to determine if the present surface finish meets the stringent roughness specifications.

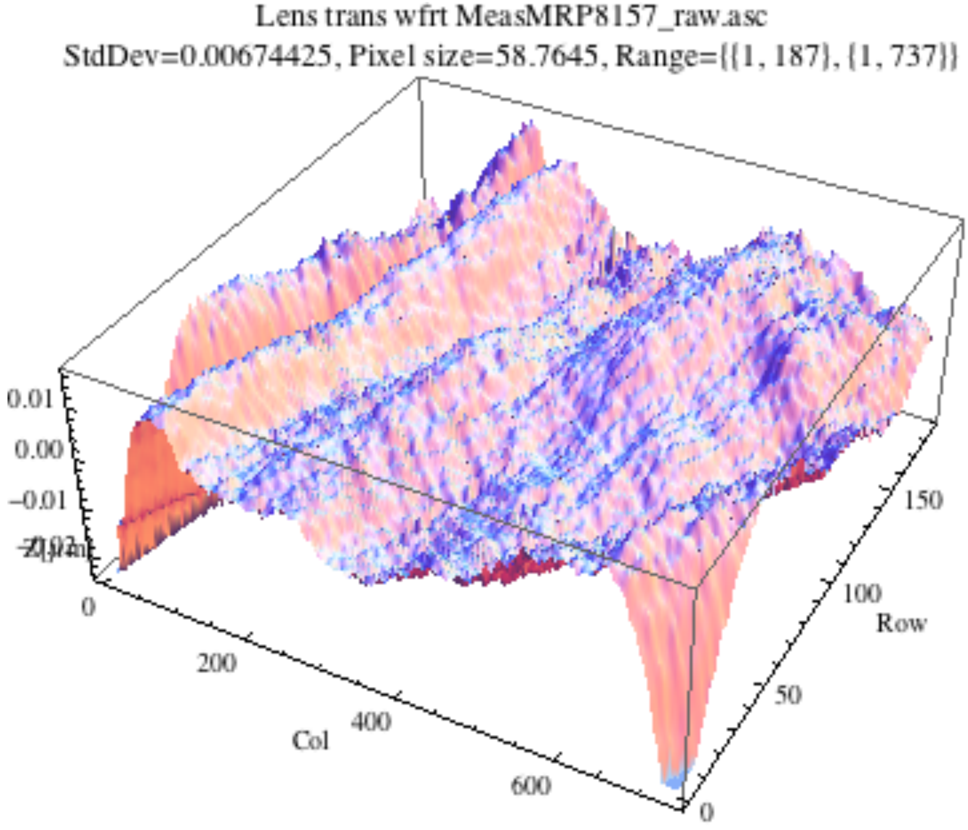
The raw data and the 3 ROI's are shown here:



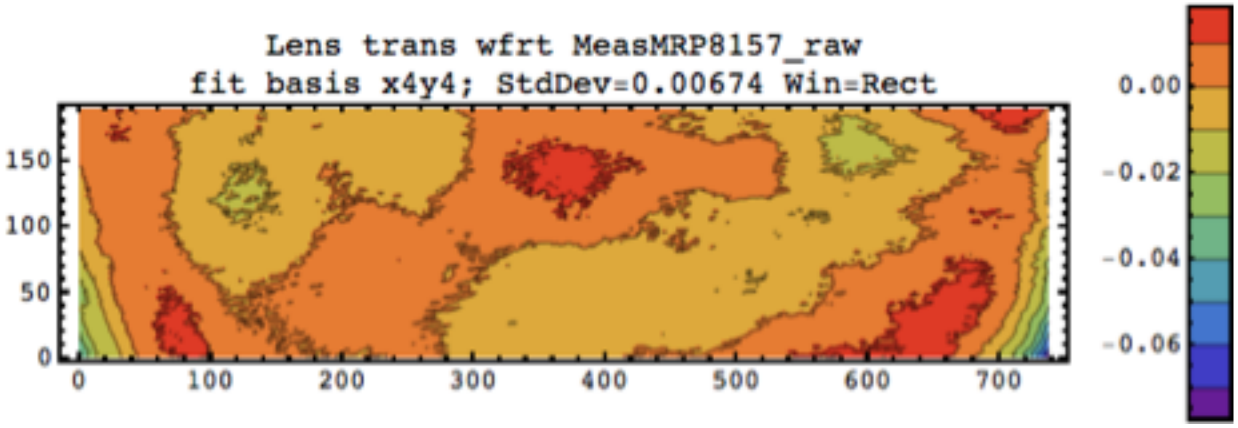
There are two main issues with this data. First, the measurement aperture is circular and does not fill the data array completely. This creates an annular region of good data points embedded in the full sensor area. Second, there is a hole in the center of the circle that contains missing data points. We need to mask the full data array and partition it into regions that exclude the missing data, both in the center and outside the circle. This requires defining several rectangular regions within the annular region that are "representative" of the true surface. We really need to test the assumption that the surface is ergodic and any subregion is statistically the same as any other. This is an implicit assumption now.

This is a measurement of the transmitted wavefront error in the lens, measured in double pass mode with a Fizeau interferometer. Presumably the transmitted beam is reflected back through the lens by a suitable reference sphere surface. Although the measurement is really of wavefront error, we call it surface error here as the error is expressed in nanometers rather than waves. The fact that the measurement is really of a lens in double pass is irrelevant here. We are only concerned with the analysis of the real data.

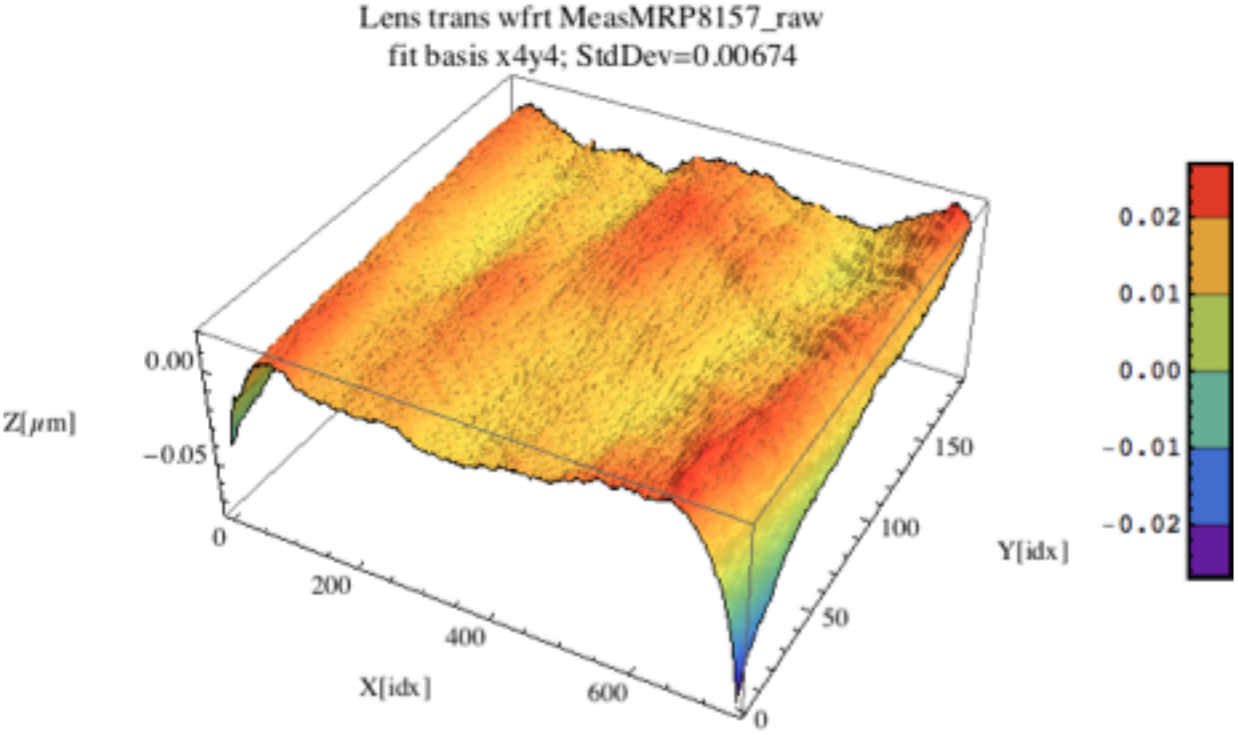
After extracting the data points from each rectangular region, the data was detrended by removing a least squares fit 4th order polynomial in x and y. This removes most of the low frequency figure error terms and leaves a residual surface that contains the higher spatial frequencies. An example of the residual surface after detrending is shown next:



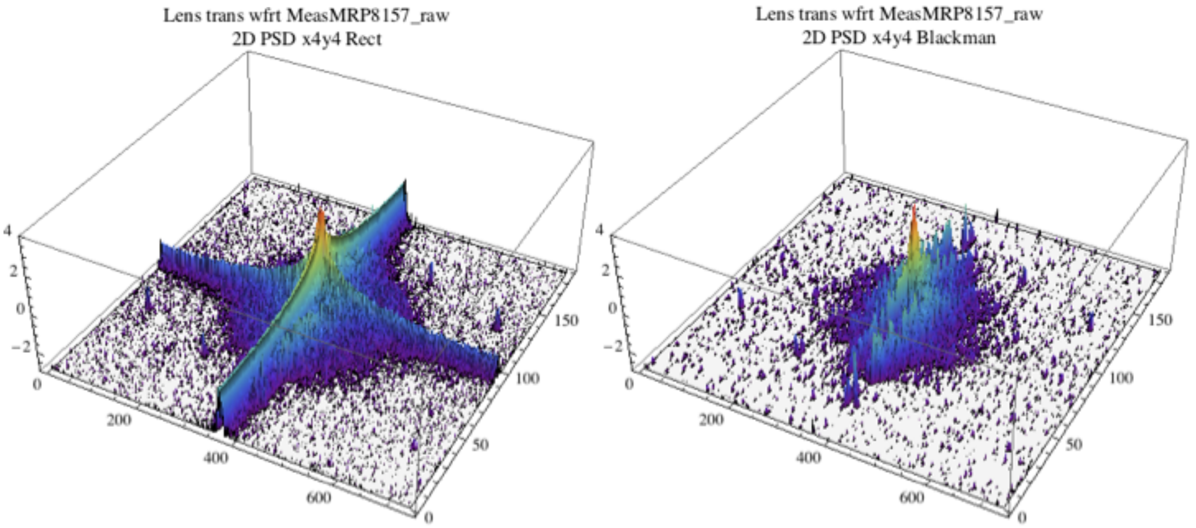
A contour plot version of this residual surface with the correct lateral aspect ratio is shown here:



The 3D surface view of this same region is shown next. Note that the aspect ratio of the 3D view is not correct. The y-axis here is stretched to fill a square box.

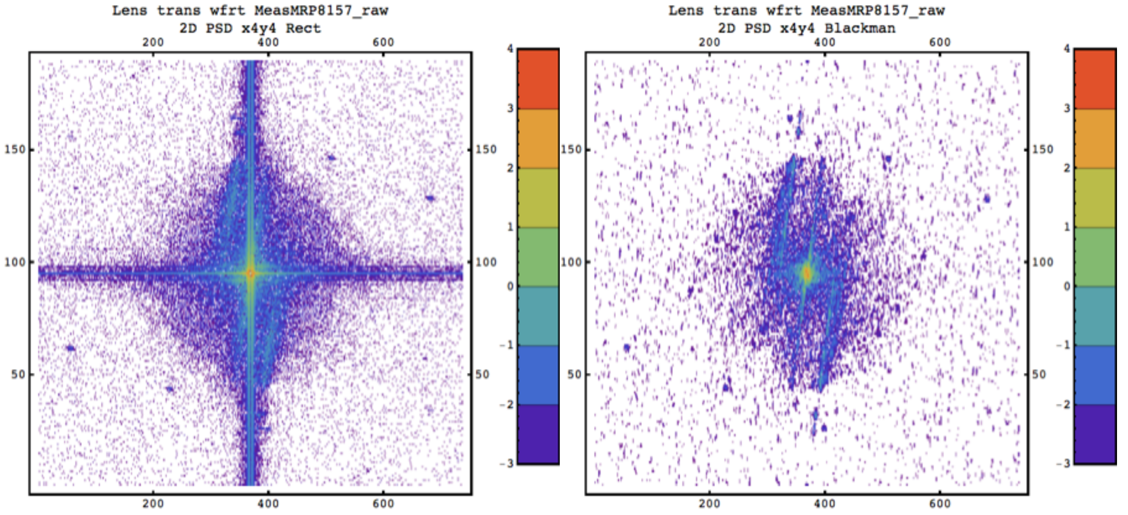


Now compute the 2D PSD for this residual surface. We do this both before and after applying a Blackman window to the data. The results are shown next in 3D plots of the log of the 2D PSD. The unwindowed ROI data can be thought of as having a rectangular window (Rect) with constant height applied to the larger surface area, identified here as a Rect window. The PSD from the Rect windowed data shows the artifacts induced by the edge discontinuities (below, left). The edges of the region do not fold over smoothly to match the other side of the array. The resultant step height discontinuity introduces the spurious power along the x and y frequency axes. By applying the Blackman window, the edge discontinuities are forced to zero and the resultant PSD is a much better estimate of the underlying “intrinsic” surface roughness (below, right).


Lens trans wfrt MeasMRP8157_raw
2D PSD x4y4 Rect

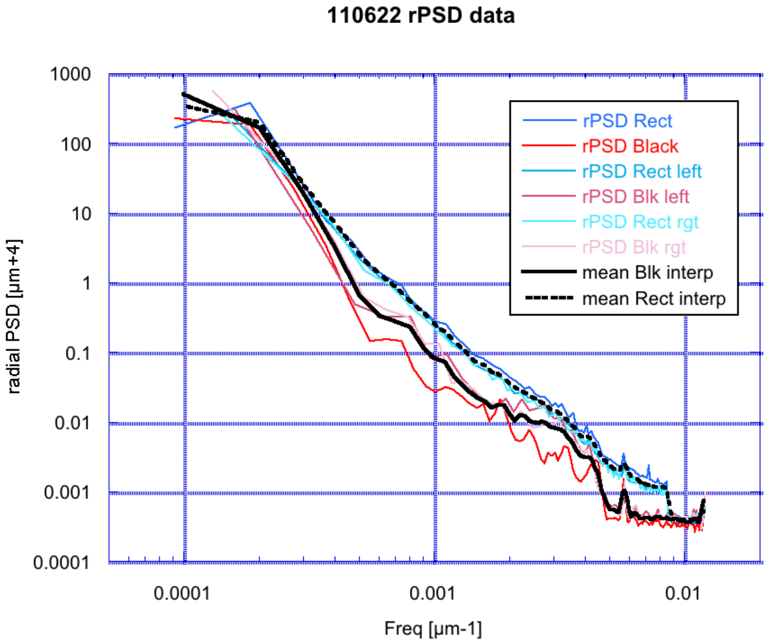
Lens trans wfrt MeasMRP8157_raw
2D PSD x4y4 Blackman

The reduction of the edge discontinuity artifacts by Blackman windowing can be seen more clearly in the 2D version of the PSD plot. The windowed data on the right clearly shows the low level power at frequencies away from the central core, with a lay that is rotated somewhat from the axes directions. The DC term is located at the center of each PSD plot and the Nyquist frequencies are located at the edges.


Lens trans wfrt MeasMRP8157_raw
2D PSD x4y4 Rect

Lens trans wfrt MeasMRP8157_raw
2D PSD x4y4 Blackman

The above PSD functions are computed by a conventional DFT in Cartesian coordinates, corresponding to the pixel array in the interferometer camera. What we are interested in is midspatial surface roughness error over a given bandwidth, over all directions. We can see from the 2D PSD on the right that the roughness has some directional component, but if we assume that it is isotropic, we can integrate over all azimuthal angles and generate an equivalent radial PSD function, which is a one-dimensional curve. This makes extracting surface statistics much easier. By performing a numerical integration, the Cartesian 2D PSD is converted into the 2D radial PSD, rPSD, shown next.



Two sets of 3 curves are shown, plus the average of the 3 as the heavy line. These curves correspond to the rPSDs for the 3 data regions, with and without the Blackman window applied. The heavy curves are generated by first interpolating each rPSD to a uniform frequency grid, then averaging the interpolated curves. The RMS is computed from each averaged curve over the indicated bandwidth.

Note that the Blackman windowed curves lie below the Rect windowed curves. This is due to the spurious power introduced into the spectrum by the edge discontinuities that are not filtered out with the Rect window. Step edge discontinuities produce excess $1/f^2$ noise, which adds to the underlying spectrum. One can see that the Rect curves follow closely a $1/f^2$ line in the spatial frequency region between 0.001 and 0.01 μm^{-1}. When this power is removed by Blackman windowing, the true surface spectrum is revealed at a lower level. One can also see in the Blackman curves a sharp drop in the spectrum around 0.0047 μm^{-1}, corresponding to a period of around 212 µm. The Nyquist frequency for the array is computed from the equivalent pixel size (depends on lateral magnification scale factor) and is given as

$$fNy = 1/(2*pix) = 1/(2*58.7) = 0.00850\ \mu m^{-1}$$

The observed drop at 0.0047 μm^{-1} is probably due to an internal aperture in the interferometer that blocks spatial frequencies higher than this value to avoid aliasing issues that would arise if the intrinsic spectrum were permitted to extend beyond the Nyquist frequency. Note that this cutoff frequency shows up clearly in the Blackman windowed curves but is hidden below the excess $1/f^2$ noise in the unwindowed spectra.

From the interpolated average curves, the RMS over the [5mm, 100µm] bandwidth is **0.74nm** for the Rect windowed data and **0.38nm** for the Blackman windowed data. This shows the necessity of proper data conditioning and windowing in computing statistical quantities from measured surface profiles. The specification for this lens is for an RMS roughness not exceed 0.5 nm. Without the Blackman window applied, the RMS roughness exceeds the limit and the lens fails. With the Blackman window applied, the RMS roughness is within the tolerance and the lens passes.