Notes on Slope and Height Nomenclature in Standards Documents as of 2009

PZ Takacs 2009

In OP1.005 – Statistical Description of Surface and Wavefront Errors, we are limiting this standard to describing errors in optical surfaces, which are by nature smooth surfaces. The bulk of previous surface texture standards deal with machined surfaces, surfaces produced by cutting tools and grinding wheels, that are intrinsically orders of magnitude rougher than high quality optical surfaces. These surfaces have structural components, such as deep grooves and peaks and valleys, that don't usually occur in finely polished optical surfaces. Traditionally, the tools available for assessing the form and roughness of these parts have been based around contact stylus instruments. The original instruments produced electronically-filtered analog data on continuous strip charts. The early standards were geared toward extracting useful parameters from these analog records. Hence, there is considerable discussion in them about defining 2RC filters and Gaussian phase-corrected filters that do not distort the transmitted profile. Filters are applied by convolution to the spatial domain data. Our approach is to do away with the mechanistic description of surface roughness and use frequency space analysis of Fourier-transformed data. The concept of bandwidth-limited statistics arises naturally from this description. Modern measuring instruments produce sampled digital data that is easily manipulated in the frequency domain by fast discrete Fourier Transform algorithms that were not common 20 years ago.

The following paragraphs extract the slope- and height-related items from specific standards documents and other supporting documents.

1. Chetwynd, D.G., *Slope Measurement in Surface Texture Analysis,* Journal of Mechanical Engineering **20**, p. 115-119 (1978).

2. Stout, K.J., Development of Methods for the Characterization of Roughness in Three Dimensions. 2000, London, UK: Penton Press. 358 pp.

3. Stout, K.J. and Blunt, L., Three-Dimensional Surface Topography. 2nd ed. 2000, London, UK: Penton Press. 285 pp.

4. Whitehouse, D.J., *Handbook of Surface Metrology*. 1994, Bristol and Philadelphia: Institute of Physics Publishing. 988 pp.

Whitehouse reference –

The Whitehouse handbook is a great reference for all kinds of surface metrology topics, but it does not give any practical recipes for slope and power spectrum issues.

Stout references -

Stout is an excellent reference for area texture characterization. He goes into great detail about digital filtering methods for 3D topography. He defines slope as a hybrid parameter in terms of finite differences of heights. He uses the symbol $\eta(x,y)$ to denote surface height and "ρ_{ij}" as the surface slope at any point {i,j} (12-10).

ISO 4287:1997 by TC57
ISO 11562:1996
ISO 3274:1996

These older standards deal with **Surface Texture: Profile method**. These give definitions of parameters for linear profiles only.

ISO 4287 predates the use of digital recording instruments and Fourier analysis methods and contains terminology that is mostly relevant to analog recording devices, such as strip chart recorders. It does not address at all how a continuous analog signal is to be sampled to produce a set of digitized data points $\{z_i\}$, where the sampling width is Δx. It does contain a simple definition of "local slope" (3.2.9) as $\frac{dz}{dx}$, the "slope of the assessed profile at a position x_i". But it gives no guidance in how to determine the x_i and, hence, does not address spatial frequency bandwidth. The definition does contain a note that the local slope, and the derived RMS quantities from it, depend "critically" on the Δx ordinate spacing, where the spacing between adjacent points parameter is determined by applying a prescribed short period cutoff filter to the continuous data. This terminology is relevant to analog scanning stylus instruments but is not relevant to modern digital data sets. The definition does give a suggested formula for computing the local slope from the filtered data:

$$\frac{dz_i}{dx} = \frac{1}{60\Delta x}\left(z_{i+3} - 9z_{i+2} + 45z_{i+1} - 45z_{i-1} + 9z_{i-2} - z_{i-3}\right)$$

based on work done by D.G. Chetwynd published in 1978, but this form has no relevance to modern digital processing Fourier transform algorithms.

The only entry for "surface slope" in the ISO Concepts Database is related to hydrologic stream flow.

Also in 4287, there is a definition of the RMS slope under "Hybrid parameters" (4.4.1) in words only without an explicit formula (!). This definition does, however, define the symbol for RMS slope in analogy to the RMS roughness symbols: $P\Delta q$, $R\Delta q$, $W\Delta q$, where the P,R,and W refer to the Profile, Roughness, and Waviness sampling lengths. These old standards were written for analog stylus profilers, not for modern digital optical instruments.

NOTE: a search for the term "surface profile" in the ISO Concepts Database fails to find the term in ISO 4287:1997. It is paragraph 3.1.4 in the document. But the search does find the term in ISO 25178-6:2010 and it refers to the source as ISO 4287:1997.

According to the ISO Concept Database, the term Power Spectral Density, PSD, appears in 3 standards: ISO 8608:1955, ISO/TS 13473-4:2008, and ISO 2041:2009 ed. 3

Note: TC57 has been disbanded. The work of this committee appears to have been taken up by TC213, which has extended the linear profile work to areal surface topography. This work program is under TC213/WG16.

It appears that all of these standards avoid the use of the terms "1D", "2D", and "3D". I don't know if this is intentional, but it eliminates the possibility for ambiguous terminology, such as "2D surface profile" that is used sometimes to mean a plot of the height as a function of one spatial coordinate, as in z(x), or height as a function of 2 spatial coordinates, as in z(x,y).

ISO 25178-2   from TC213

The currently-active ISO TC213 committee website has an extremely useful document: "List of terms used in GPS standards and in standards relevant for GPS standards" http://isotc213.ds.dk/213N471.pdf   which contains a list of all General Product Specification terms and the standards in which they are defined, dated January 2002. This committee deals with developing GPS standards, which includes surface topography, roughness, and form standards. They are responsible for the 25178 series of standards that deal with areal surface texture. However, nowhere in this list does the term "slope" appear.

But, in ISO/DIS 25178-2-"Terms, definitions and surface texture parameters" dated 2008-04-02, the term "gradient" is introduced as a hybrid parameter in definition 3.2.7 "local gradient vector" as $\left(\frac{\partial z}{\partial x},\frac{\partial z}{\partial y}\right)$ where z=z(x,y) is the "ordinate value" ≡ "height of the scale-limited surface at position (x,y)". This definition derives the local slope from the measured height. Further on in 25178-2 the RMS area gradient is defined in the usual way: $Sdq=\sqrt{\frac{1}{A}\iint\left[\left(\frac{\partial z(x,y)}{\partial x}\right)^2+\left(\frac{\partial z(x,y)}{\partial y}\right)^2\right]dxdy}$, where the symbol for the RMS gradient is defined as Sdq, a 3-component symbol related to the 2-component symbol Sq for the height roughness.

ISO 25178 introduces the filter terms S, L, and F that indicate boundaries in spatial period space for 3 regions: S-filter for small scale, L-filter for large lateral scale, and F-operator for form. These designations are extremely confusing. Why is F an "operator" and not a "filter" like the other two? S removes shorter periods, but L and F remove longer periods. Why not just let the user specify what spatial period range is desired and forget about forcing it into the SLF form? These designation apparently supersede the profile filter parameters in 4287-97: λc, λf, and λs defined for the analog stylus instruments.

ISO 25178 does everything in the spatial domain. Our description will be in frequency space and will make use of the Power Spectral Density function (PSD). There is no definition for PSD in 25178, nor in any of the GPS standards, according to the list of terms document. For instance, in 25178- Part 602 – "*Nominal characteristics of non-contact (confocal chromatic probe) instruments*", there is no mention of instrument transfer function in frequency space. Everything is described in spatial terms.

ASME B46.1-2002 & 2009

B46 uses Z(x) to denote the height profile and Z(x,y) to denote the areal height. The profile roughness statistics are denoted by the usual Ra, Rq, etc. nomenclature. Slope is not defined as a distinct surface property but is indirectly defined in the "Hybrid Parameters" section as $\frac{dZ}{dx}$. It is spelled out explicitly for sampled data in terms of the 7-point formula of Chetwynd (defined above) and is given the symbol "Δ". Slope is a derived quantity here. The analogues of the Ra and Rq parameters for slope are given by RΔa and RΔq. Areal parameter analogues to the profile parameters are denoted with the letter "S" in place of "R", i.e. Sa and Sq, as in the 25178 document. Waviness is denoted by the letter "W", with only the *waviness height*, Wt explicitly defined for profiles and *area waviness height,* SWt, defined for area data.

In the 2009 version, there is no symbol for the areal slope, but there is a parameter that defines the "area root mean square directional slope", *Sdq(Θ)*, analogous to the linear slope *RΔq*. But there is an inconsistency in the symbology: Why is the areal slope not denoted as *SΔq(Θ*), with the Δ symbol in place of *"d"?*

Section 1.4.4 defines the PSD for a linear profile in the generic analytic continuous-variable form and also for sampled digital data. It also defines APSD to be the PSD over an area, again giving the continuous analytic form and the digital form for sampled data. These definitons appears to be taken directly from Stout's book.

Filters to separate profile data into roughness, waviness, and form errors are described only in the spatial domain, not in frequency space,

ISO/TS 16610-20:2006(E) - *Geometrical product specifications (GPS) — Filtration —Part 20: Linear profile filters: Basic concepts*

The ISO 16610 series documents deal with specifying various filters to apply to profile data in the spatial domain to get bandwidth-limited filtering by convolution. There is a section on "Transfer function" that touches on Fourier transforms, but only in terms of explaining how the cut-off frequency arises from a particular spatial domain Gaussian convolution filter. It does not make the next logical step of using spectral analysis to transform the profile first into the frequency domain and then apply a simple frequency filter and then transform back into the spatial domain to obtain the filtered profile. By restricting filtering to the spatial domain, the definition, construction, and use of convolutional filters is extremely complicated.

Note that the List of Terms on the TC213 website is dated 2002. It does not have any of the new terms that appear in the 16610 documents, like "convolution" or "transfer function", so relying on the list without checking newer standards since 2002 may cause one to overlook more recent terminology. The newer standards documents that have been prepared in electronic form are searchable PDF files; the older standards documents that have been scanned in from paper copies are not searchable. This makes it easy to find a particular term in a collection of new standards that reside in a particular computer folder, but older documents will not be scanned.

None of these filter documents discuss slope.

SEMI MF1811-0704 - Guide For Estimating the Power Spectral Density Function and Related Finish Parameters from Surface Profile Data

SEMI 1811 was published in 2004 and contains most of the detailed definitions and recipes for computing the PSD function from linear profiles. It was written originally by Gene Church a number of years before and was shepherded through the standards process by John Stover. It is based upon the work done by Church in collaboration with Stover and Takacs in connecting surface roughness measurements with actual scattered light measurements with normal incidence visible light and grazing incidence x-rays. It is the starting point for OP1.005.

This document uses the symbol lower-case "*m(x)*" to indicate the profile slope and upper-case "Z(x)" to indicate profile height. The profile RMS roughness is denoted by "Rq" and RMS slope by "Δq".

This document deals with surface slope on an equal footing with surface height, since there are profiling instruments that measure surface slope directly, rather than surface height. It gives recipes for computing RMS height and slope error quantities from both height and slope domain PSD functions. It is particularly useful in that Church takes great pains to make sure Parseval is satisfied by keeping track of all proper normalization factors in the transformation equations.

Symbol for PSD

B46 uses PSD and APSD for the profile and area PSD symbols, respectively.
The SEMI standard use S_1 and S_2 for the profile and area PSD symbols.
B46 and 25178 uses S for Surface, with added qualifiers, so S is not a viable candidate.
Whitehouse use P for the PSD but he gives no recipe for the periodogram calculation.

Symbol for slope

B46 uses Δ_i
SEMI use m(x)

I propose the letter "G" be used to denote slope. G(x) denotes profile slope; G(x,y) denotes area slope. This puts slope on a equal footing with height, denoted by "Z", when slope is measured directly by a slope-measuring profiler. The symbol "Δ" for slope has the connotation of it being a derived quantity, a difference between two heights, rather than a stand-alone single quantity. "G" has no such baggage attached and follows logically from the name "Gradient". RMS slope error would be denoted "Gq".

Note that G(x) can be derived from Z(x) by Fourier transform techniques, and vice versa, with out the need to define a finite difference formula method. If one has a band-width limited measurement of height or slope, one can transform between slope and height domains very easily.

[1] K. R. Freischlad, and C. L. Koliopoulos, "Modal Estimation of a Wave-Front from Difference Measurements Using the Discrete Fourier-Transform," Journal of the Optical Society of America A, 3(11), 1852-1861 (1986).

uses vector **S** as the gradient of the wavefront, with Z as the symbol for the set of complex expontntials as basis functions over the rectangular aperture.

[1] D. L. Lessor, J. S. Hartman, and R. L. Gordon, "Quantitative surface topography determination by Nomarski reflection microscopy. I. Theory," J. Opt. Soc. Am., 69(2), 357-366 (1979).

Lessor uses symbol ψ as the surface tilt designator.

[1] A. H. Schooley, "A Simple Optical Method for Measuring the Statistical Distribution of Water Surface Slopes," J. Opt. Soc. Am., 44(1), 37-40 (1954).

Schooley use symbol α as surface slope designator.

[1] C. S. Palm, R. C. Anderson, and A. M. Reece, "Laser probe for measuring 2-D wave slope spectra of ocean capillary waves," Appl. Opt., 16(4), 1074-1081 (1977).

Palm (1977) use θ as ocean surface slope designator.

[1] Q. Li, M. Zhao, S. Tang et al., "Two-dimensional scanning laser slope gauge: measurements of ocean-ripple structures," Appl. Opt., 32(24), 4590-4597 (1993).

Li use T as the ocean surface tilt angle symbol.

z symbol for height in ISO 17769:2008