Rotationally Invariant Polynomials:

These polynomials must have $m=0$.

The set of rotationally invariant polynomials that exist in the current set of 36 Zernike terms described in ISO 10110-5, Appendix A.3, first edition are as follows:

$Z_{2,0}$ is $Z_3(r,\alpha)2r^2-1$
$Z_{4,0}$ is $Z_8(r,\alpha)6r^4-6r^2+1$
$Z_{6,0}$ is $Z_{15}20r^6-30r^4+12r^2-1$
$Z_{8,0}$ is $Z_{24}70r^8-140r^6+90r^4-20r^2+1$
$Z_{10,0}$ is $Z_{35}(r,\alpha)252r^{10}-630r^8+560r^6-210r^4+30r^2-1$

These polynomials have no azimuthal dependence and describe the wavefront aspheric approximation f_{wri} in section 3.2.6. Additional rotationally invariant terms can be added to more accurately approximate the wavefront.

Rotationally Varying Polynomials:

These polynomials must have $|m|>0$.

Rotationally varying polynomials have a radial function as well as a sine or cosine function which dictates the order of the azimuthally varying function.

These polynomials come in pairs, having an equal value for n and the positive/negative equal integer walue for m.

For example:
$Z_{2,2}$ is astigmatism about 0 degrees. $r^2\cos(\alpha)$
$Z_{2,-2}$ is astigmatism about 45 degrees. $r^2\sin(\alpha)$

A vector summation along with the arc tangent expression will give the magnitude and the directio of the resultant pair of matching polynomials.

$$Directional\ Angle=1/m*\arctan\{(Z_{n,(-m)}/Z_{n,m})\}$$

$$Magnitude=2*\sqrt{(Z_{n,(-m)})^2+(Z_{n,(m)})^2}$$

The Zernike Polynomial Term is written:
$Z_{n,m}$ where: **n** = radial order
m = azimuthal order
n is an integer > 0.
n - |**m**| must be a non-negative even integer including 0.
If **n** = 8; then **m** can equal 8, 6, 4, 2, -2, -4, -6, -8 only.

Definitions:

$Z_{n,m}$ is the coefficient value for the expression:

$R_{nm}(r) \times \sin|(m)|(\alpha)$ for m is a negative number

$R_{nm}(r) \times \cos m(\alpha)$ for m is a positive number

And further: $R_{nm}(r)$ is the radial function:

$$R_{nm}(r) = \sum_{s=0}^{l} (-1)^s \frac{(n-s)!}{s!(l-s)!(n-l-s)!} r^{n-2s}$$

where $l = \frac{n-m}{2}$

$\sin|m|(a)$ and $\cos m(a)$ are the azimuthal functions

Example:

$Z_{8,-4}$ (pronounced, Z eight minus four) would be the magnitude of the Zernike coefficient for the following equation.

$$28r^8 - 42r^6 + 15r^4 (\sin 4\alpha)$$

Which is an 8th order radial function and 4th order azimuthal function.